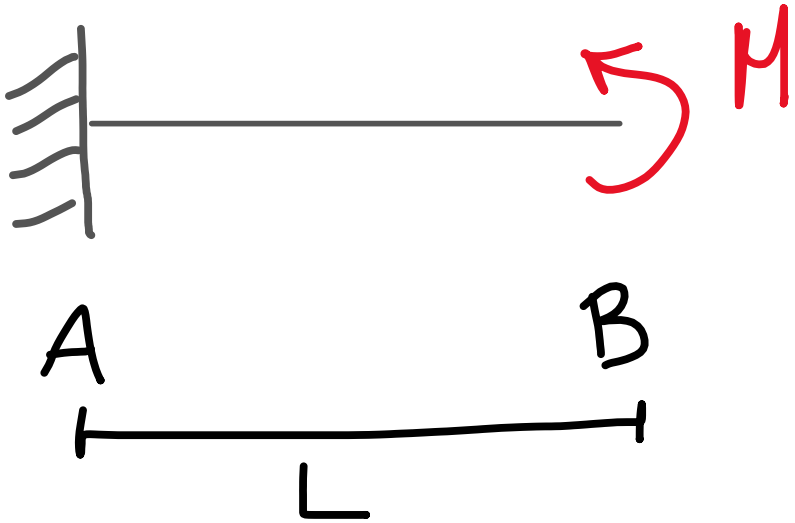
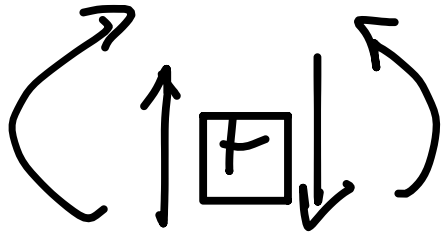
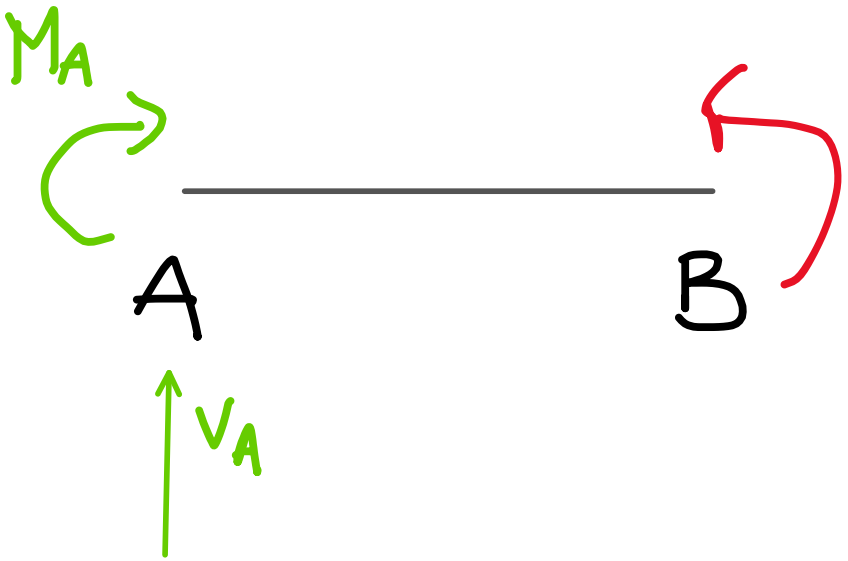
Seminario esami di stato
II sessione 2021
aula civile

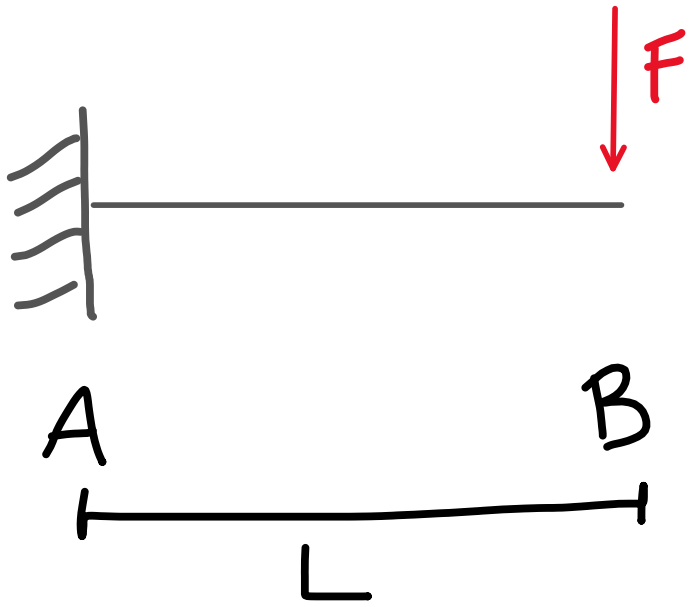




$$\sum F = 0$$

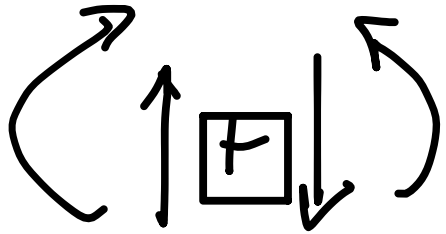
$$\sum M = 0$$



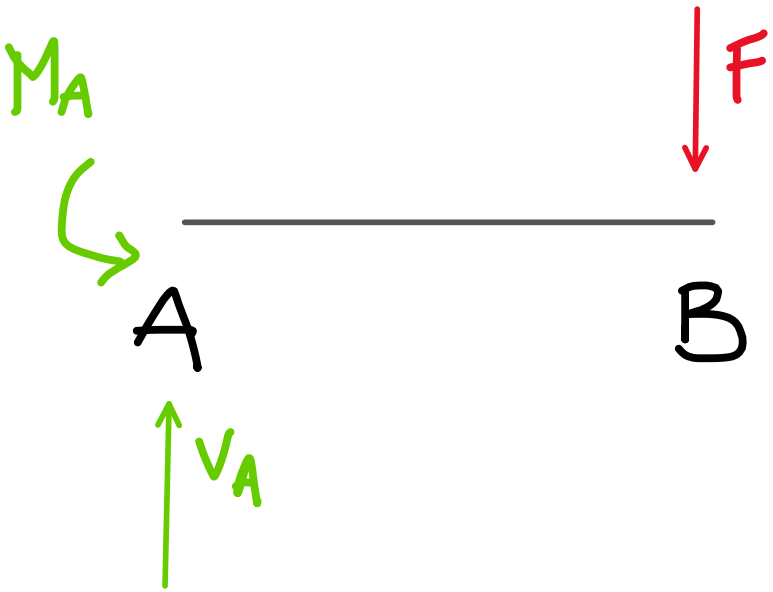


$$\sum F = 0 \qquad \frac{dT}{dx} = -q(x)$$

$$\sum M = 0 \qquad \frac{dM}{dx} = T(x)$$

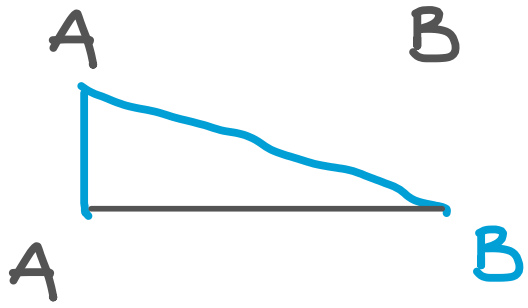


$$V_A - F = 0 \qquad V_A = F$$

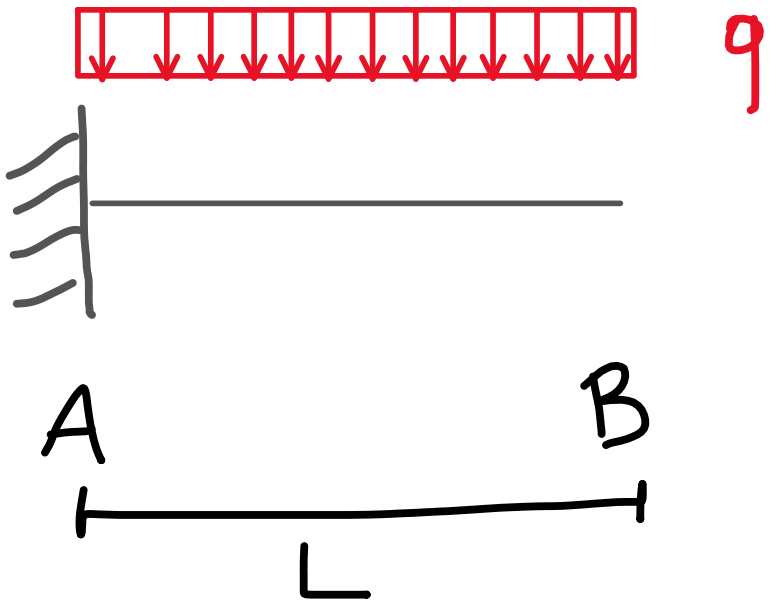


$$(B)\ M_A - V_A \cdot L = 0 \qquad M_A = F \cdot L$$

$T = \text{cost} = F$



$M_A = FL \qquad M_B = 0$

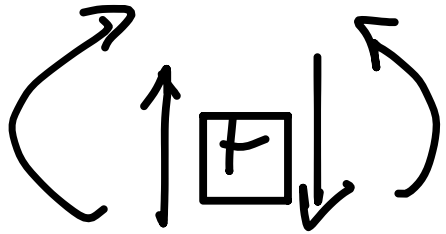


$$\sum F = 0$$

$$\sum M = 0$$

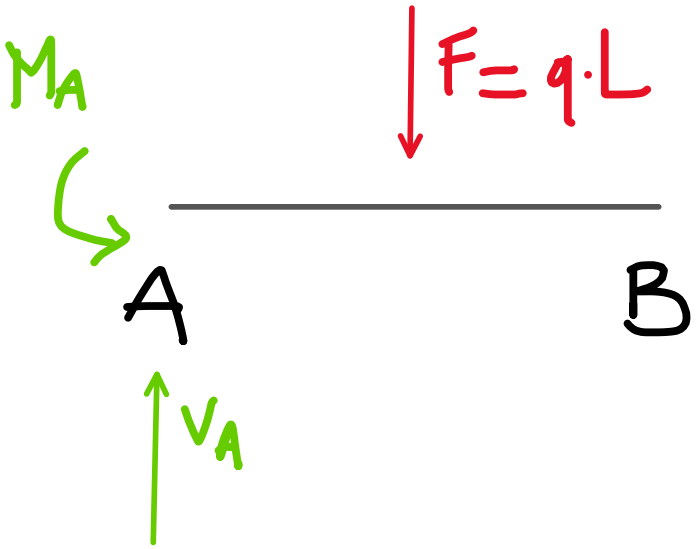
$$\frac{dT}{dx} = -q(x)$$

$$\frac{dM}{dx} = T(x)$$

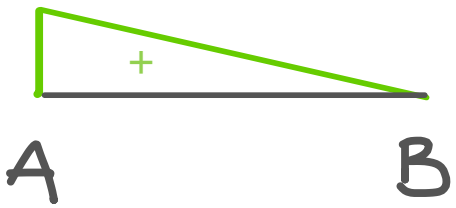


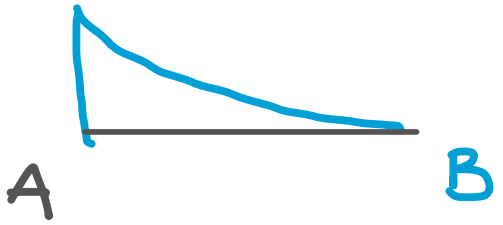
$$V_A - F = 0 \qquad V_A = F = q \cdot L$$

$$(B)\ M_A - V_A \cdot L + q \cdot \frac{L}{2} = 0 \qquad M_A = q\frac{L^2}{2}$$

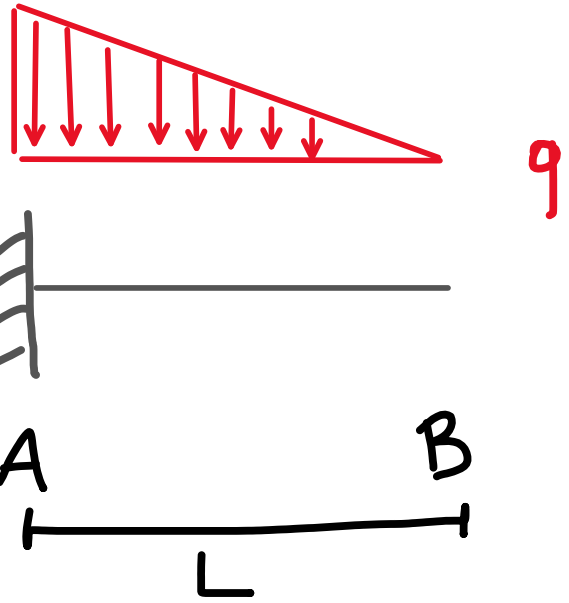


$$T_A = q \cdot L$$

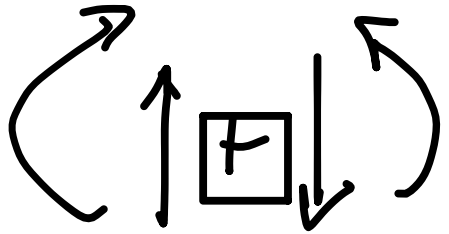




$$M_A = q\frac{L^2}{2} \qquad M_B = 0$$

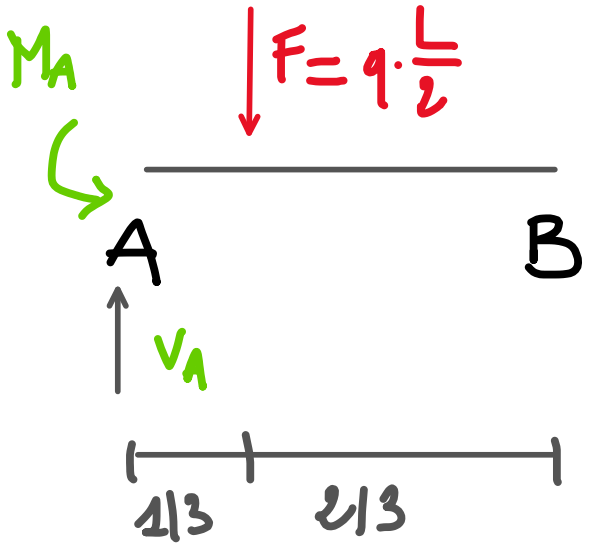


$$\sum F = 0$$
$$\sum M = 0$$

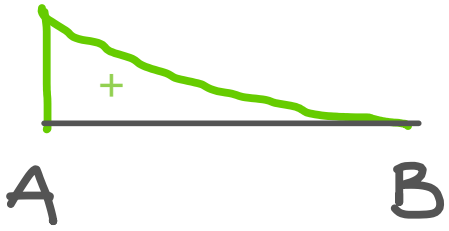


$$V_A - F = 0 \qquad V_A = F = q \cdot \frac{L}{2}$$

B) $M_A - V_A \cdot L + q \cdot \frac{L}{2} \cdot \frac{2}{3} L = 0$ $\qquad M_A = \frac{1}{6} q L^2$

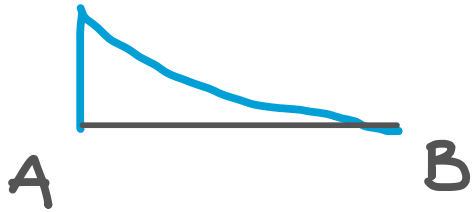


$$T_A = q \cdot \frac{L}{2}$$

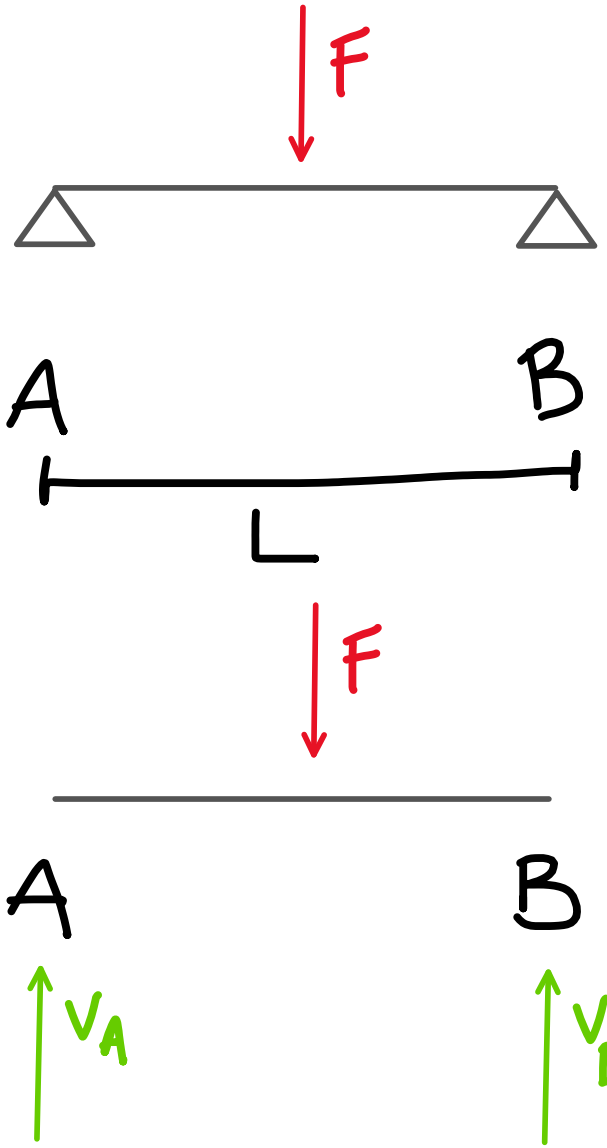


$$\frac{dT}{dx} = -q(x) \qquad q(x) = \frac{q_{MAX}}{L}(L - x)$$

$$\frac{dM}{dx} = T(x)$$

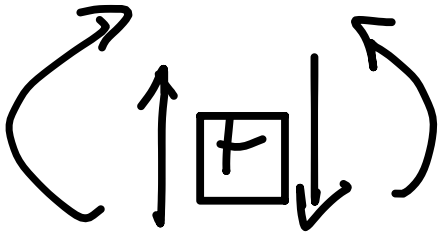


$M_A = \frac{1}{6} q L^2 \qquad M_B = 0$



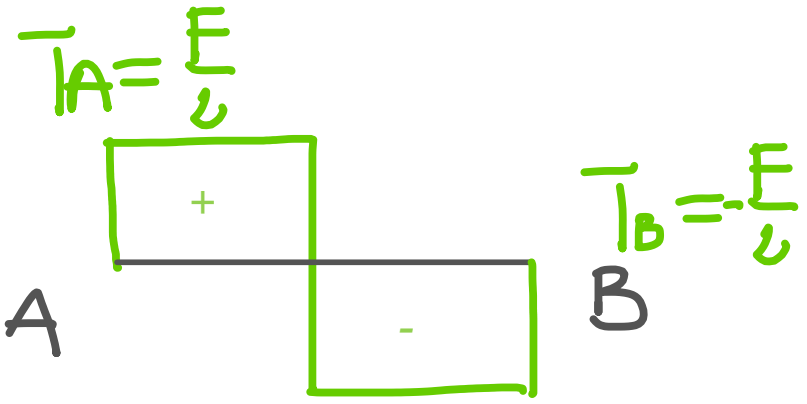
$$\sum F = 0$$

$$\sum M = 0$$

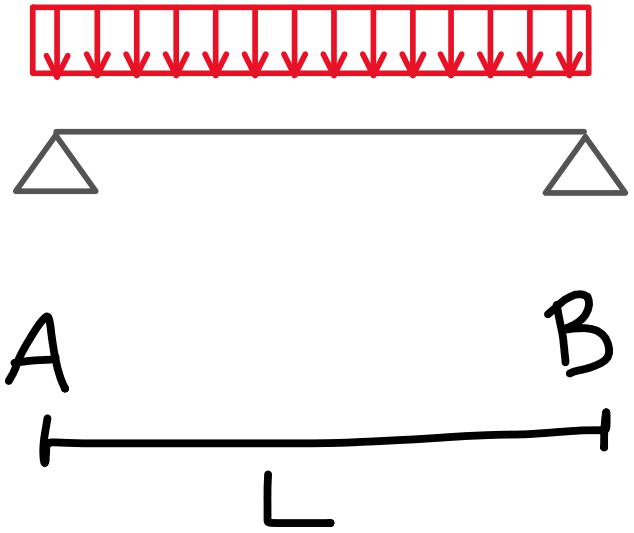


$$V_A + V_B - F = 0 \qquad V_B = \frac{F}{2}$$

$$(B)\ V_A L - F \cdot \frac{L}{2} = 0 \qquad V_A = \frac{F}{2}$$



$M_A = 0$ $\qquad$ $M_B = 0$

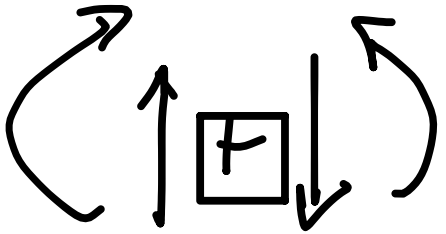


$$\sum F = 0$$

$$\sum M = 0$$

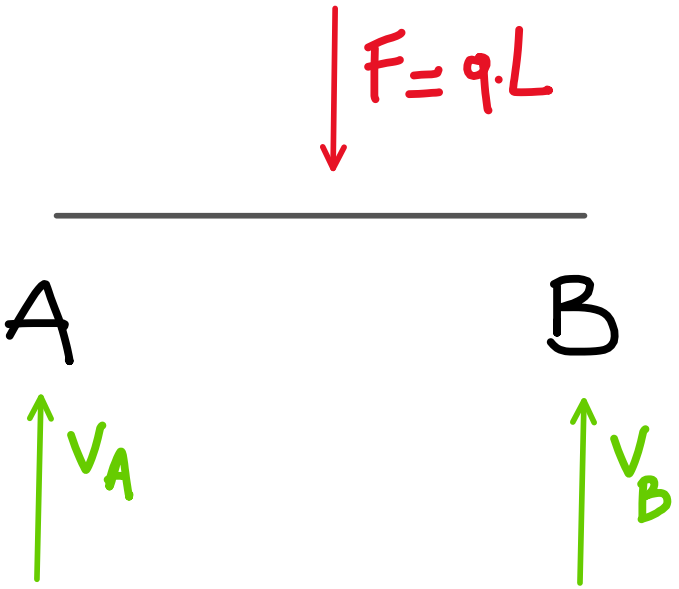
$$\frac{dT}{dx} = -q(x)$$

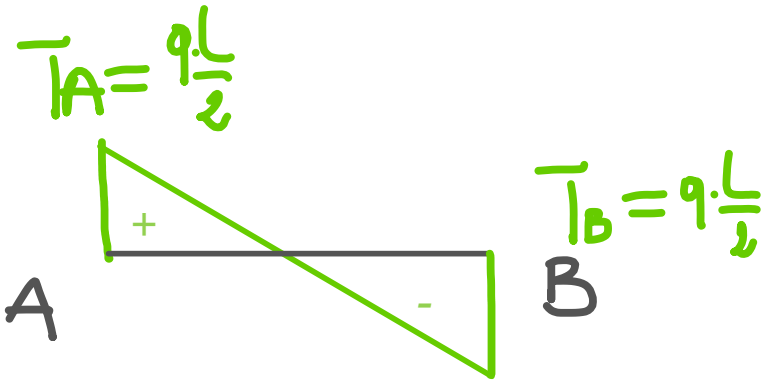
$$\frac{dM}{dx} = T(x)$$

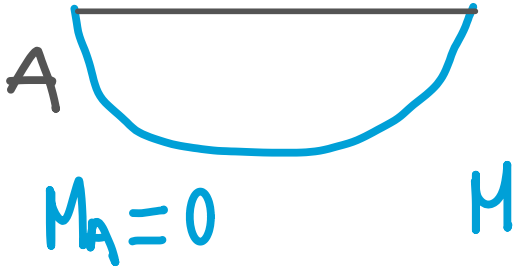


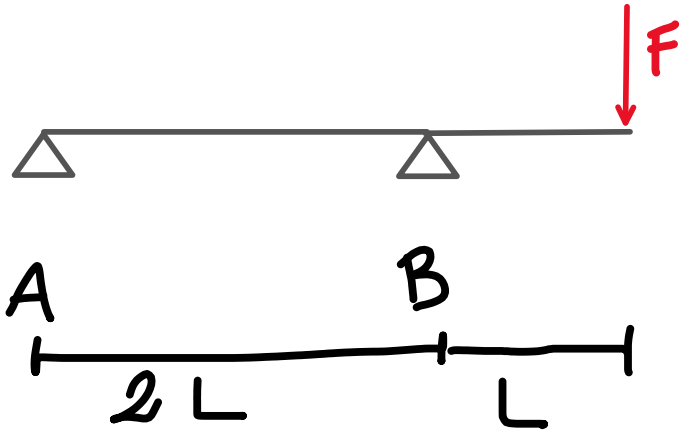
$$V_A + V_B - F = 0 \qquad V_B = \frac{F}{2} = \frac{qL}{2}$$

$$(B)\ V_A L - F\frac{L}{2} = 0 \qquad V_A = \frac{F}{2} = \frac{q \cdot L}{2}$$







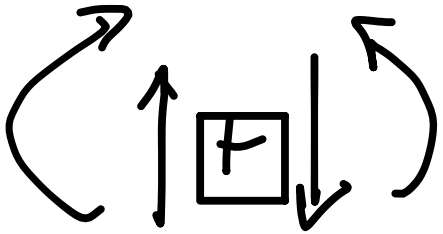


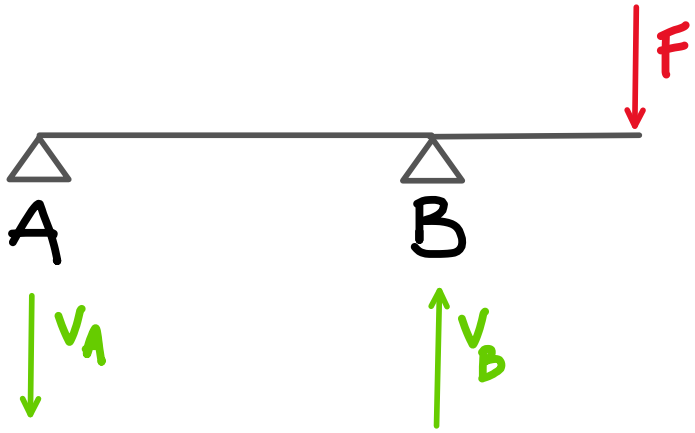
$$\sum F = 0$$

$$\sum M = 0$$

$$\frac{dT}{dx} = -q(x)$$

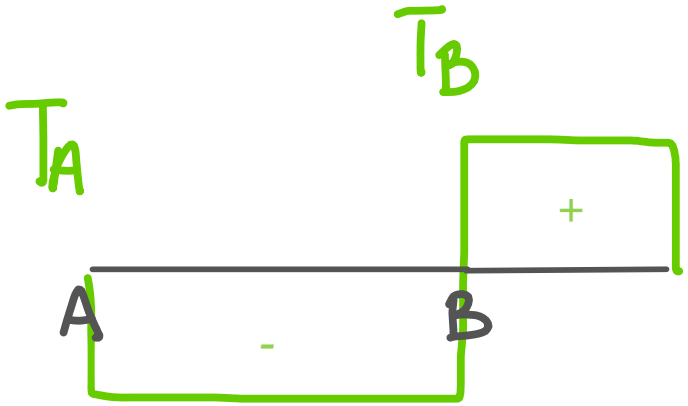
$$\frac{dM}{dx} = T(x)$$

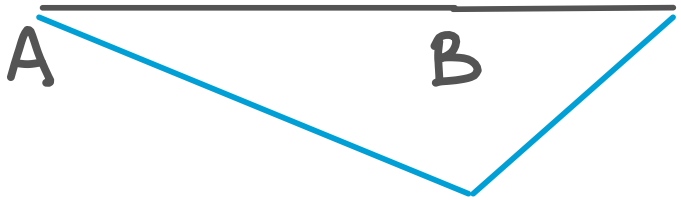


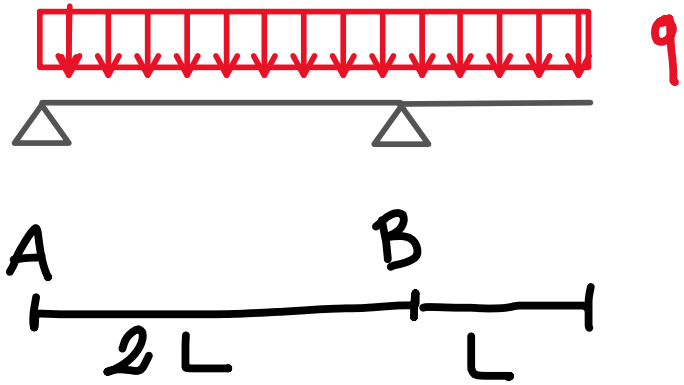


$$V_A + V_B - F = 0 \qquad V_A = \frac{F}{2}$$

$$(A)\ V_B \cdot 2L - F \cdot 3L = 0 \qquad V_B = \frac{3}{2}F$$





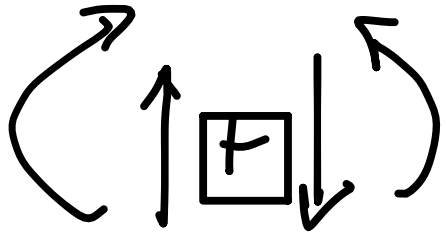


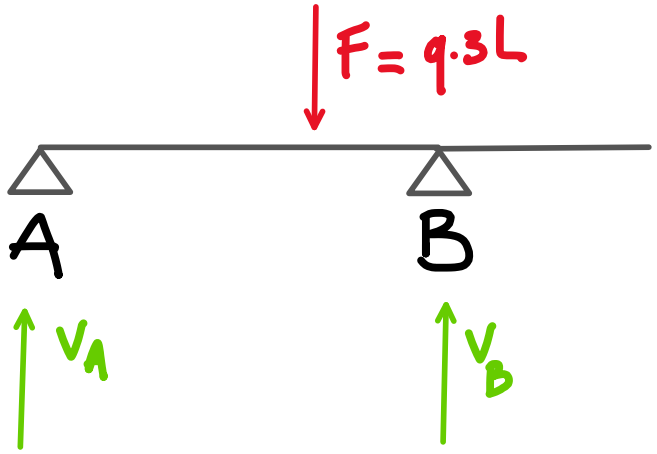
$$\sum F = 0$$

$$\sum M = 0$$

$$\frac{dT}{dx} = -q(x)$$

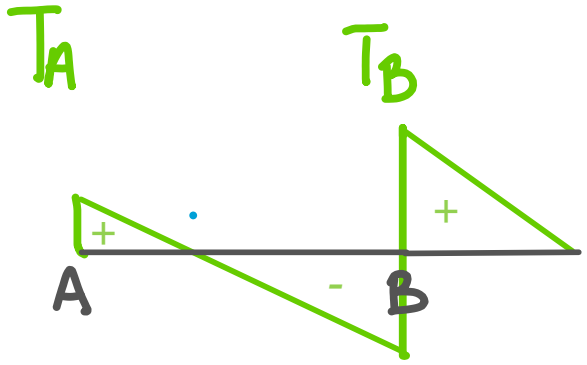
$$\frac{dM}{dx} = T(x)$$

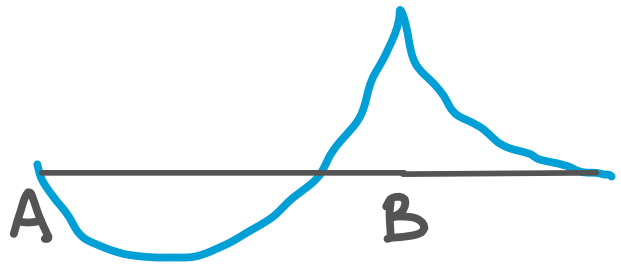




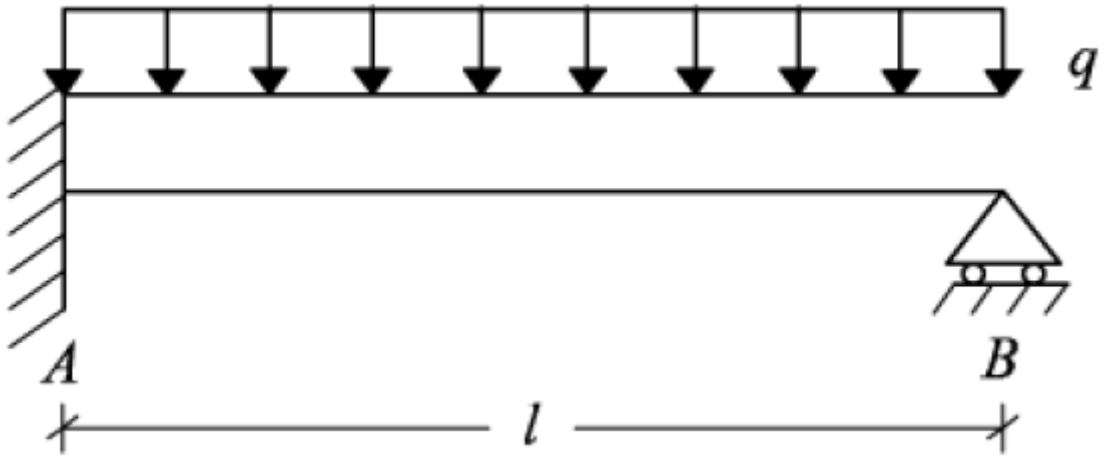
$$V_A + V_B - F = 0 \qquad V_A = \frac{F}{4}$$

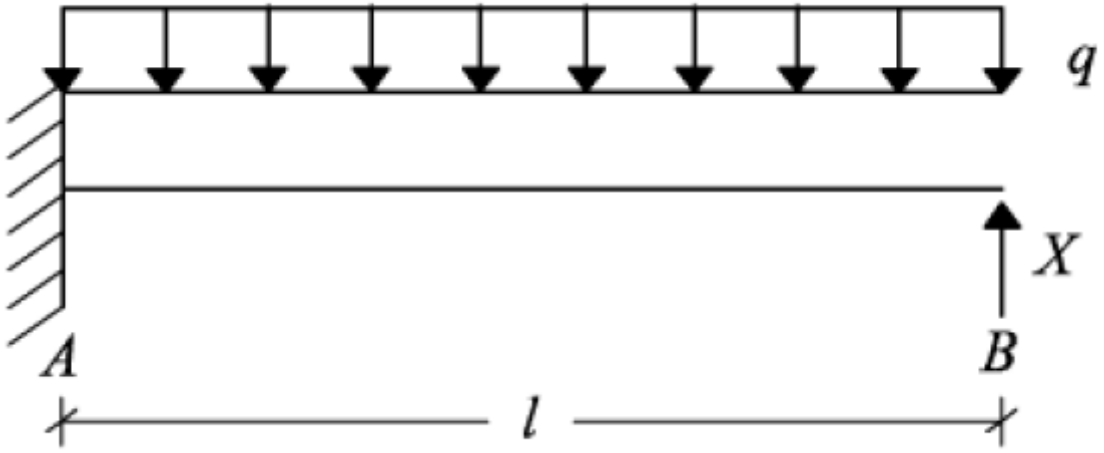
$$(A)\ V_B \cdot 2L - F \cdot \frac{3}{2}L = 0 \qquad V_B = \frac{3}{4}F$$



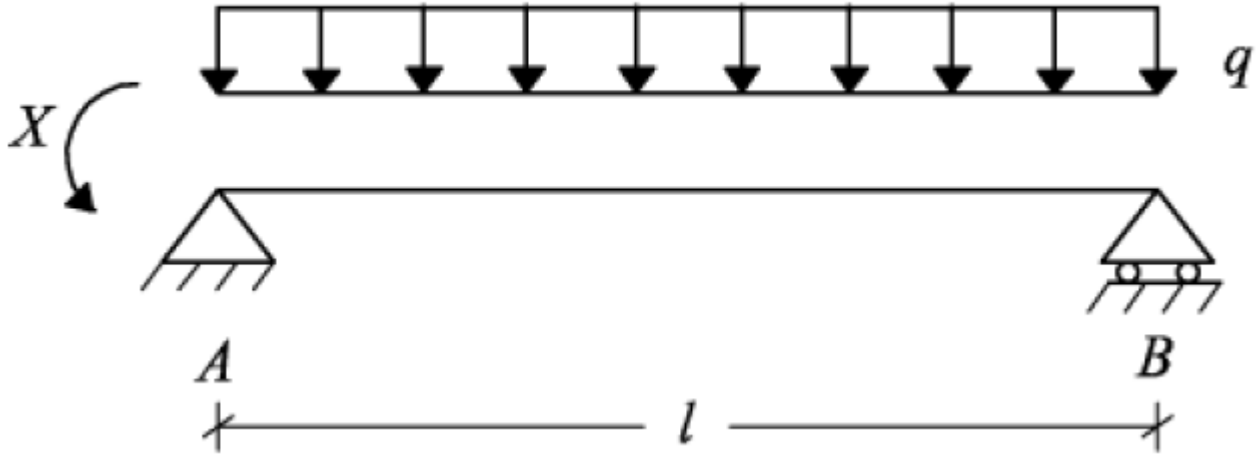


Esempio di struttura iperstatica

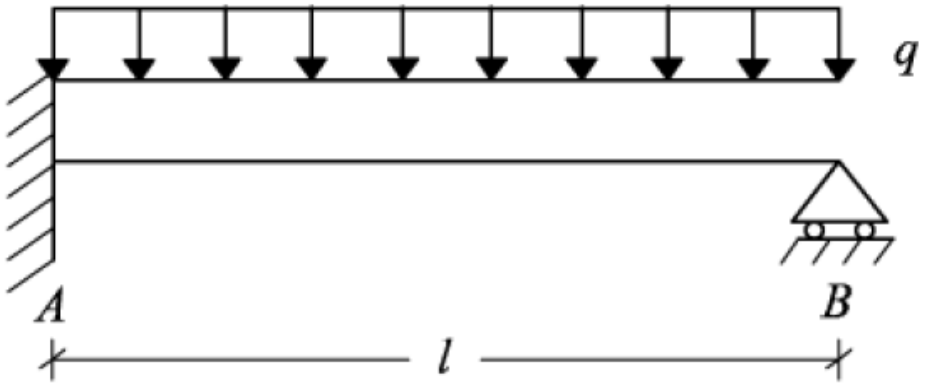


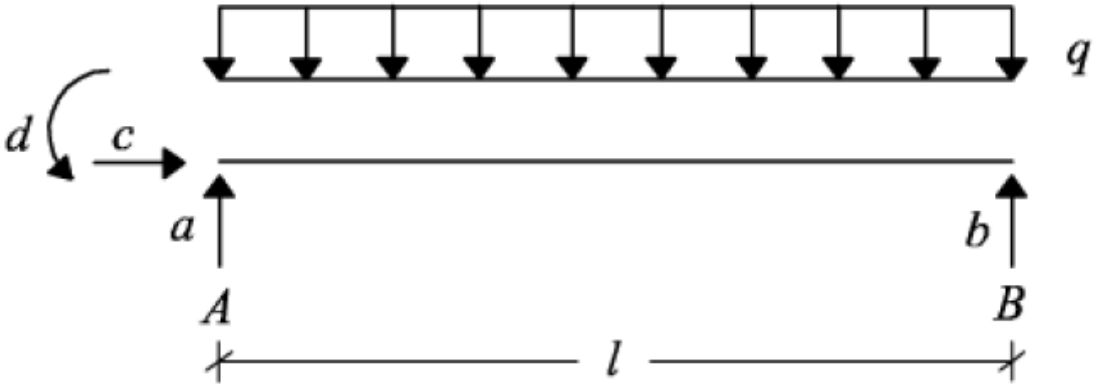


Struttura isostatica di riferimento a)



Struttura isostatica di riferimento b)

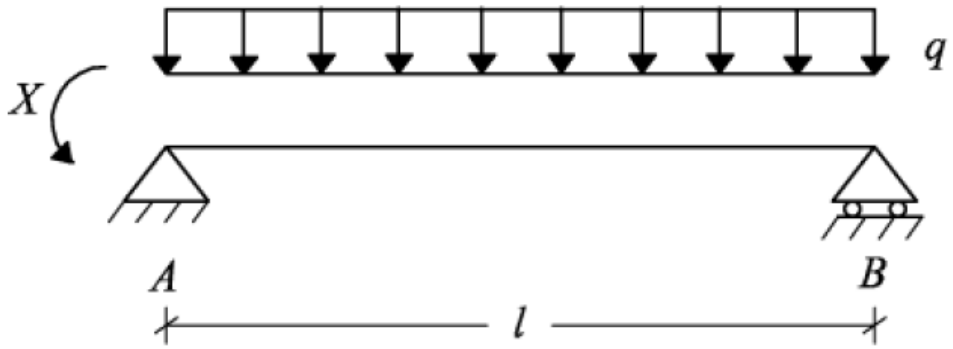




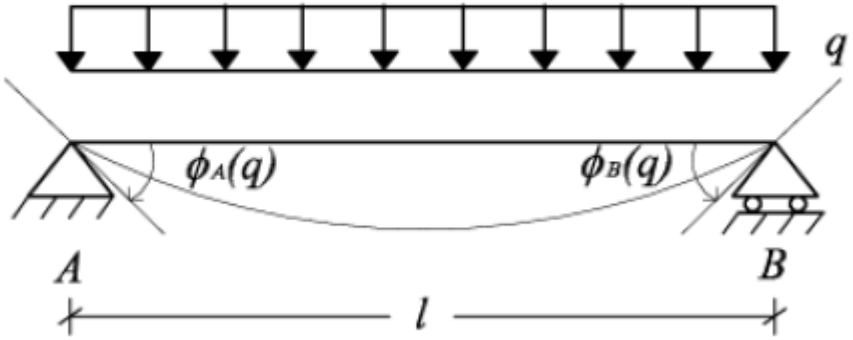
Struttura una volta iperstatica: grado di vincolo è *v=4* mentre i gradi di libertà sono *l=3 (v-l=4-3=1*)

$$\begin{cases} c = 0 \\ a + b = ql \\ d + bl - \dfrac{ql^2}{2} = 0 \end{cases}$$

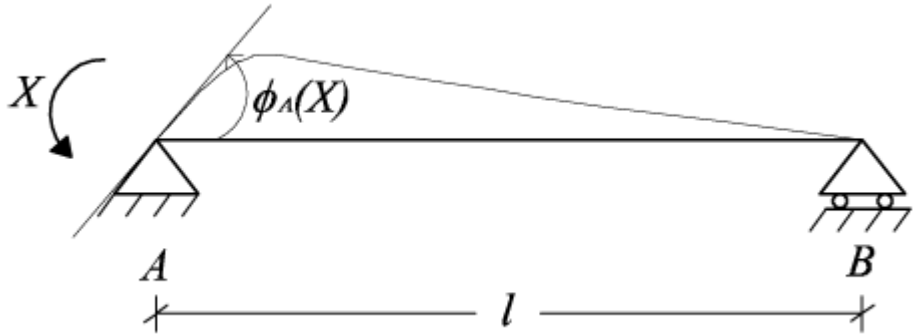
1) *scegliere la struttura isostatica di riferimento*



2) *valutare l'effetto cinematico provocato dai carichi attivi sulla struttura isostatica*



3) imporre la congruenza della deformazione e determinare il valore dell'incognita/e iperstatica



$$\varphi_A := \varphi_A(X) + \varphi_A(q) = 0$$

I risultati in termini di rotazioni (determinabili ad esempio, con l'equazione della linea elastica):

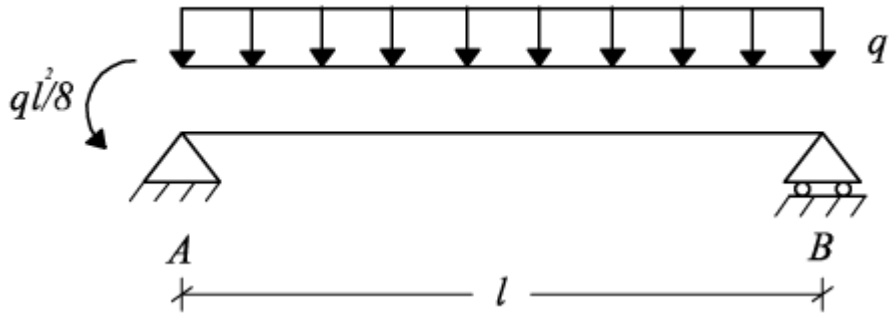
$$\varphi_A(X) = \frac{Xl}{3EJ} \qquad \varphi_A(q) = -\frac{ql^3}{24EJ}$$

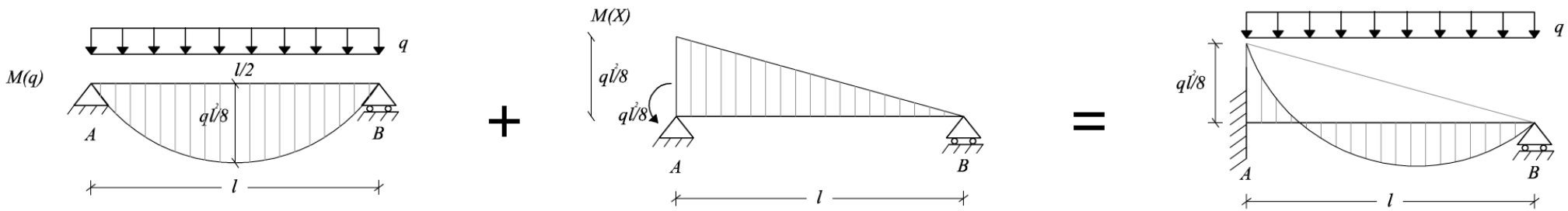
$$\varphi_A := \varphi_A(X) + \varphi_A(q) = 0$$

$$\frac{Xl}{3EJ} - \frac{ql^3}{24EJ} = 0$$

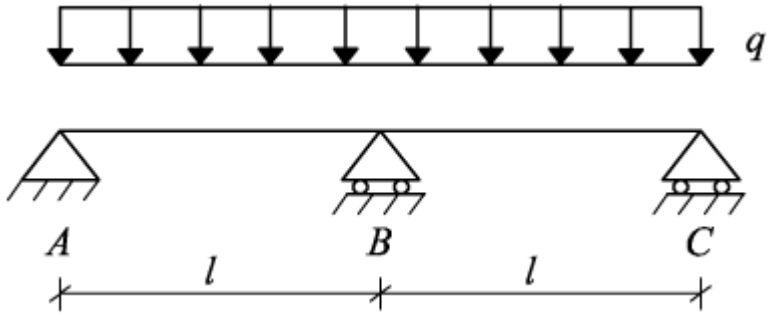
$$X = \frac{ql^2}{8}$$

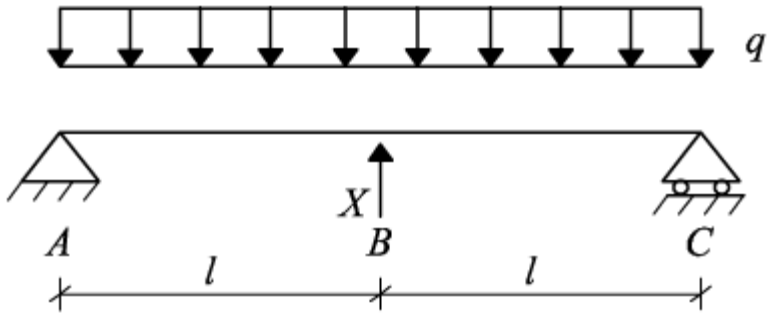
4) determinazione dei diagrammi delle sollecitazioni sulla struttura iperstatica



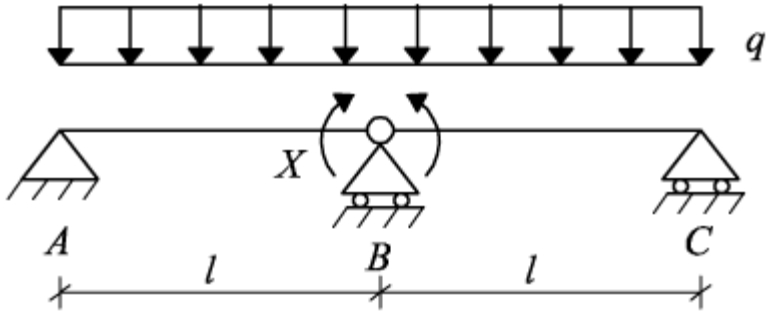


Trave continua su tre appoggi



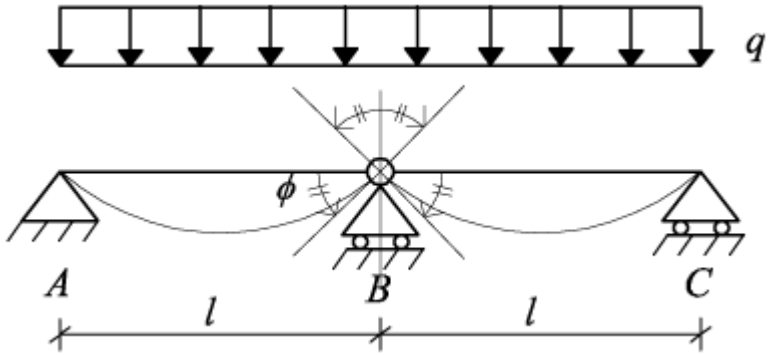


Struttura isostatica di riferimento a)

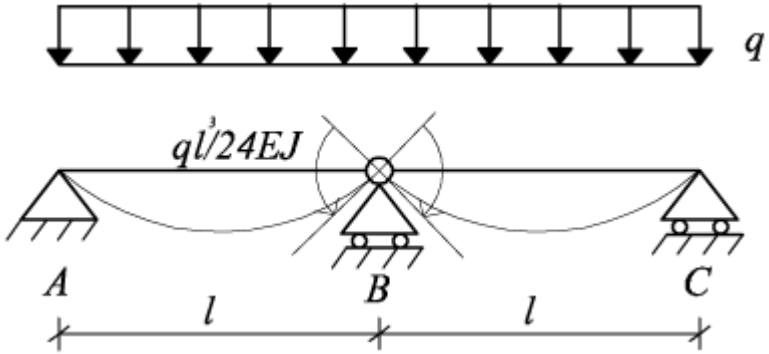


Struttura isostatica di riferimento b)

Deformazione della struttura isostatica di riferimento per effetto del carico distribuito



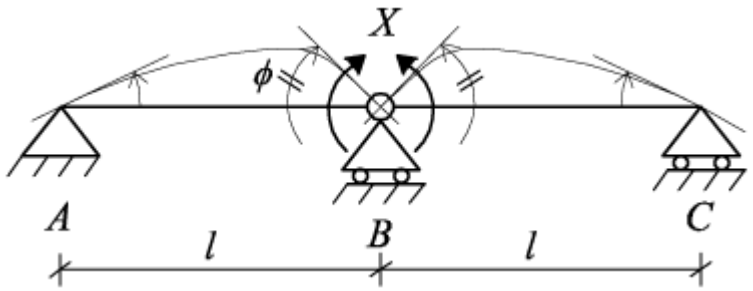
il valore assoluto della rotazione della sezione sull'appoggio per una trave doppiamente appoggiata soggetta ad una carico distribuito pari a *q*



Il valore della rotazione relativa prodotta da *q* sarà dunque pari a:

$$\Delta\varphi_B(q) = \varphi_{BS}(q) - \varphi_{BD}(q) = ql^3/(24\,EI) - (-ql^3/(24\,EI)) = \boxed{\frac{ql^3}{12EJ}}$$

L'incognita iperstatica *X* dovrà produrre una rotazione relativa uguale in valore assoluto ed opposta in segno rispetto a quella prodotta dalla densità di carico, ottenendo come effetto la deformata mostrata nella seguente figura:



Ricordando che per una trave doppiamente appoggiata soggetta ad un momento *X* applicato in una delle due sezioni terminali, la rotazione della sezione dove *X* è applicato vale *Xl*/(*3EJ*), il valore della rotazione relativa prodotta da *X* nella sezione B sarà dunque pari a:

$$\Delta\varphi_B(X)=\varphi_{BS}(X)-\varphi_{BD}(X)=\boxed{\Delta\varphi_B(X)=-\frac{Xl}{3EJ}-\left(-\frac{Xl}{3EJ}\right)=-\frac{2}{3}\frac{Xl}{EJ}}$$

Imponendo il vincolo di continuità:

$$\boxed{\Delta\varphi_B := \varphi_B(q)+\varphi_B(X)=0} \quad \Rightarrow \quad \frac{ql^3}{12EJ}-\left(-\frac{2}{3}\frac{Xl}{EJ}\right)=0\rightarrow X=\frac{ql^2}{8}$$

È possibile determinare i diagrammi del momento flettente della struttura iperstatica, sommando per sovrapposizione degli effetti i diagrammi dovuti rispettivamente alla densità di carico e alla coppia *X* nella struttura isostatica di riferimento.